# Tensor Networks, Entanglement, and Geometry

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1607.05753 with John McGreevy

It from Qubit

Simons Collaboration on Quantum Fields, Gravity and Information

# Is quantum many-body physics “hard”?

- Experimental answer: obviously yes!
- Complexity answer: yes, because we can encode hard problems into quantum many-body physics – BQP, QMA, NP, glasses, and all that
- Hopeful theorist’s answer: certainly in some cases, but perhaps not in many (still poorly understood) cases of physical interest?
- By “easy” I mean: a problem that can be solved in polynomial time on a classical computer, caveats: physical intuition/understanding?, unfavorable polynomial scaling?

# Two potentially "easy" classes?

- Ground states or thermal states of quantum field theories
  - Why? Renormalization group structure of entanglement → efficient tensor network representations (for regulated field theory)

[Vidal "MERA", BGS-McGreevy "s-sourcery", …]

- Non-equilibrium steady states
  - Why? Coupling to environment → decoherence, low entanglement, local thermal equilibrium [Prosen-Znidaric, many others …]
  - Today: a systematic quantum information approach to construct efficient classical representations of such states (and a connection to gravity)

[BGS-McGreevy 1607.05753,
Mahajan-Freeman-Mumford-Tubman-BGS 1608.05074]

# Thermal states

$$\rho = \frac{e^{-H/T}}{Z}$$

$$H = \sum_x H_x$$

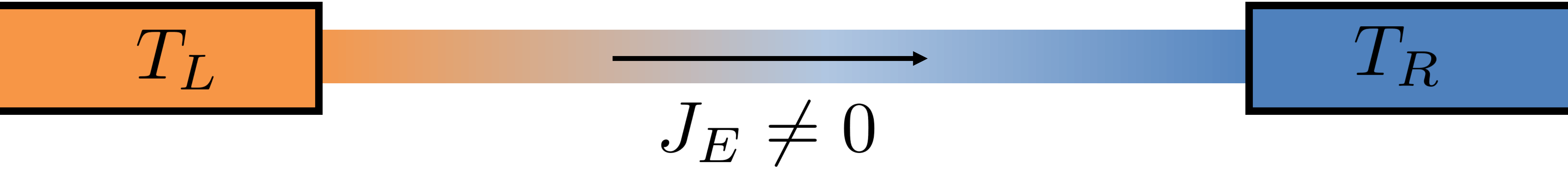
Local in space, few-body, translation invariant

- Typically have short-range correlations (except classical critical points)
- Area law for mutual information [Wolf-Cirac-Hastings-Verstraete]
- Always "trivial" for sufficiently high temperature

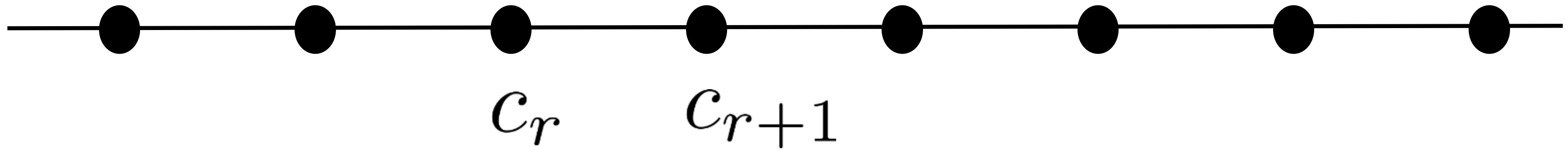
# "Hydrodynamic" states

$$\rho \propto \exp\left(-\sum_x \frac{H_x}{T_x} + \ldots\right)$$

- Local thermal equilibrium $\xi \nabla T_x \ll T_x$
- Generically carry currents, e.g. charge and heat



# Non-interacting fermions




$$\{c_r, c_{r'}^\dagger\} = \delta_{r,r'}$$

$$H = -w \sum_r \left[ c_r^\dagger c_{r+1} + c_{r+1}^\dagger c_r \right]$$

solve using momentum eigenstates $$H = \sum_k \epsilon_k c_k^\dagger c_k \qquad \epsilon_k = -2w \cos k$$

thermal physics $$\langle c_r^\dagger c_{r'} \rangle_T \sim e^{-|r-r'|/\xi} \qquad \xi = a/T$$

# Fermion thermal circuit

- Intuitively, the fermion thermal state should be easily preparable
- Construction: [BGS-McGreevy 1607.05753]
  - Introduce second copy of system
  - Find two-copy gapped Hamiltonian whose ground state is a purification of the original thermal state
  - Adiabatically evolve down from infinite temperature

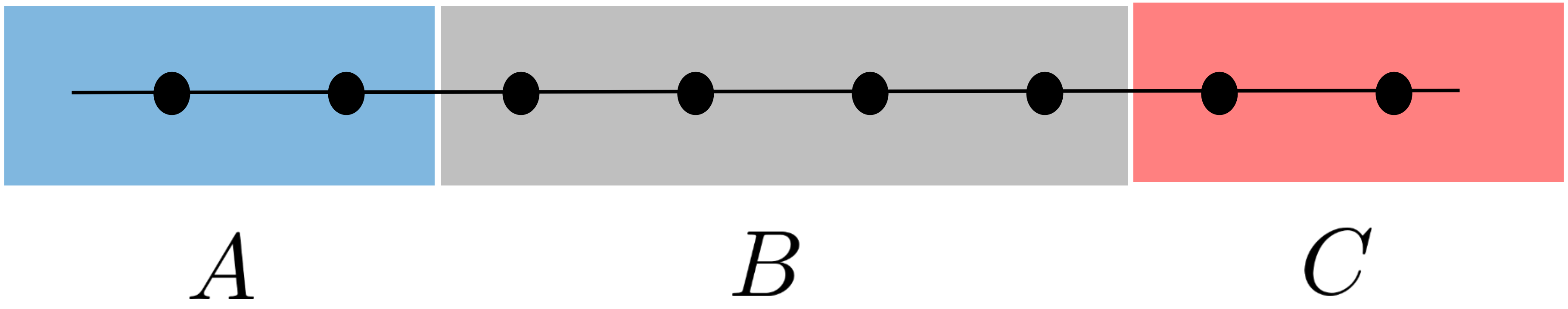
$$H_T = \sum_k \left\{ (1-2f_k) c_k^\dagger c_k - (1-2f_k)\tilde{c}_k^\dagger \tilde{c}_k - 2\sqrt{f_k(1-f_k)}(c_k^\dagger \tilde{c}_k + \tilde{c}_k^\dagger c_k) \right\}$$

interaction range in real space is the thermal length

$$H(\eta) = H_{T/\eta}$$

family of Hamiltonians, gapped, bounded range, infinite T ground state is product

# What about the fermion system generalizes?




$$S(A) = -\text{Tr}(\rho_A \log \rho_A)$$

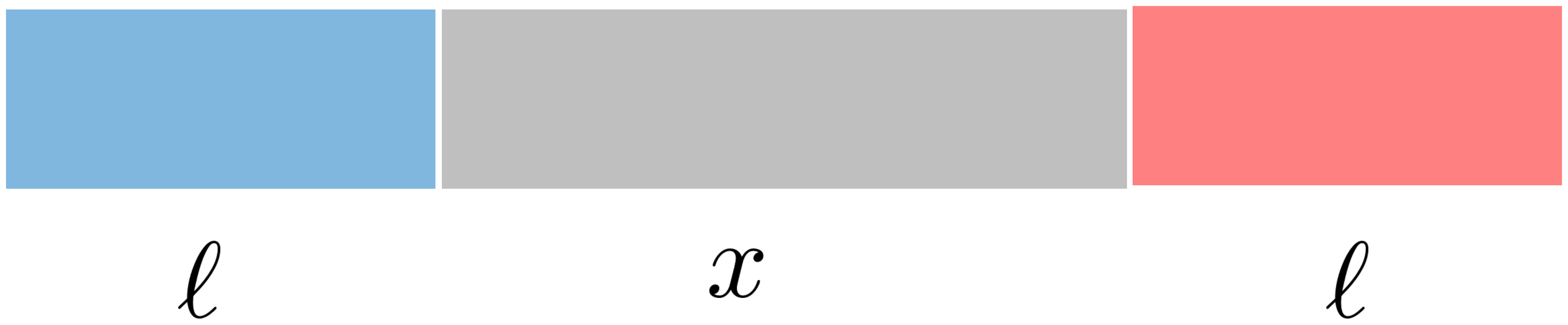
$$I(A:C|B) = S(AB) + S(BC) - S(B) - S(ABC)$$

conditional mutual information

$$I(A:C|B) \approx 0$$ if B is large enough

[BGS-McGreevy 1607.05753]

# Small CMI generalizes: conformal field theory




$$I(A:C|B) = \frac{c}{3}\log\left(\frac{\sinh^2[\pi T(\ell+x)]}{\sinh[\pi T x]\,\sinh[\pi T(2\ell+x)]}\right)$$ exact for any CFT in 1+1d

$$I(A:C|B) \sim c e^{-\pi T x}$$ large x

[BGS-McGreevy 1607.05753]

# Reconstruction from small CMI

- When the conditional mutual information is zero, there is an exact reconstruction map that depends only on $\rho_{AB}$ and $\rho_{BC}$

$$\rho_{ABC} = \rho_{BC}^{1/2} \rho_{B}^{-1/2} \rho_{AC} \rho_{B}^{-1/2} \rho_{BC}^{1/2}$$ [Petz]

- When the conditional mutual information is small, there is an approximate reconstruction map that depends only on $\rho_{AB}$ and $\rho_{BC}$

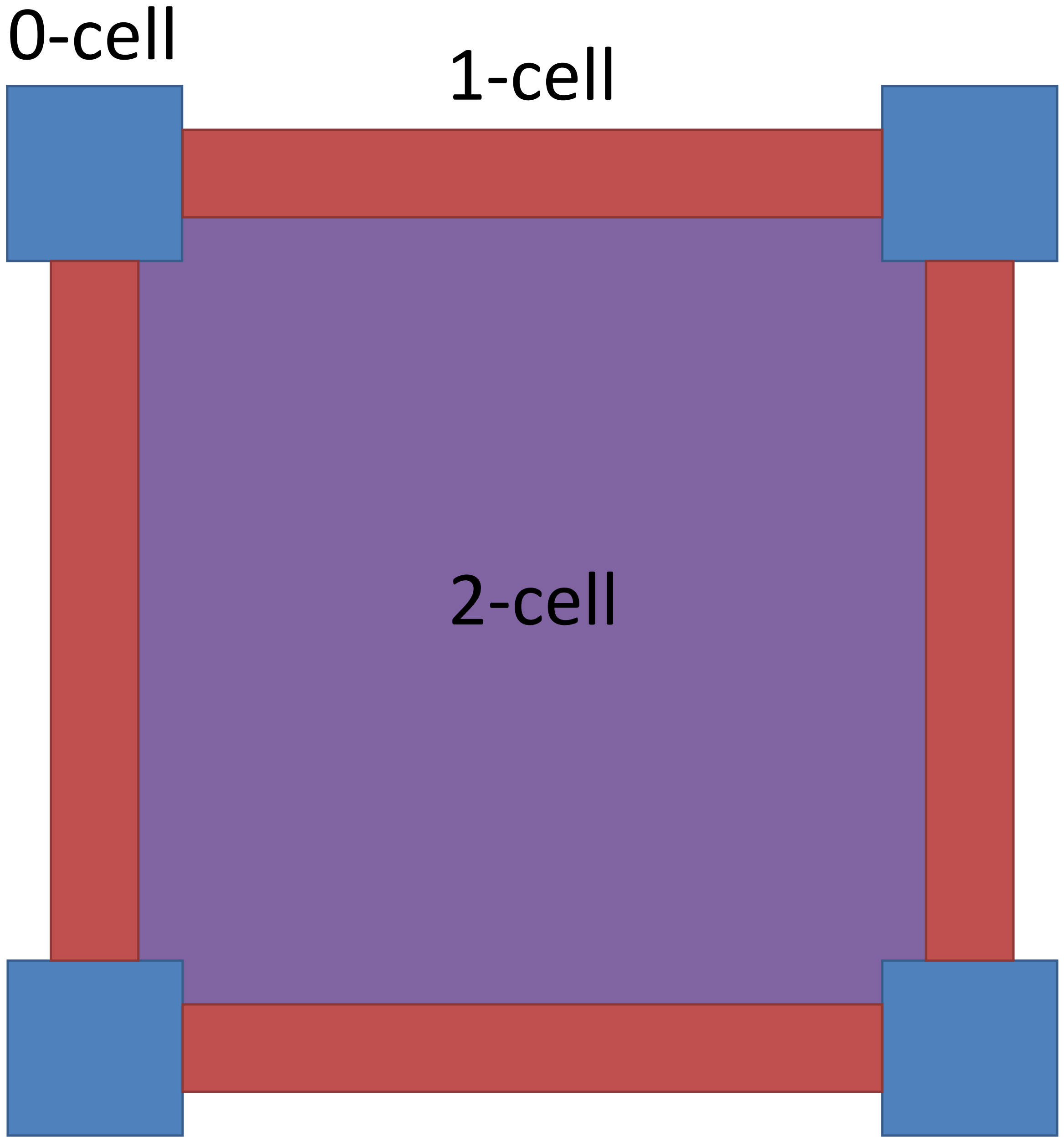
[Fawzi-Renner, Sutter-Fawzi-Renner, Wilde…]

- Essentially, we are gluing local together to produce the global state

very much like hydrodynamics

# General entropic argument

- For many quantum field theories (and quantum lattice models) at non-zero temperature, we expect the CMI of appropriate regions to approximately vanish
- Entropy S = [Volume Terms] + [Area Terms] + [Small corrections]
- Can be explicitly checked for a wide variety of models and is always valid at sufficiently high temperature, can fail for topological models e.g. 4d toric code / 2-form gauge theory
- Main result: Combining CMI reconstruction results with a cellular construction shows that suitable thermal states can be efficiently prepared by short-range quantum channel [BGS-McGreevy 1607.05753]


0-cell
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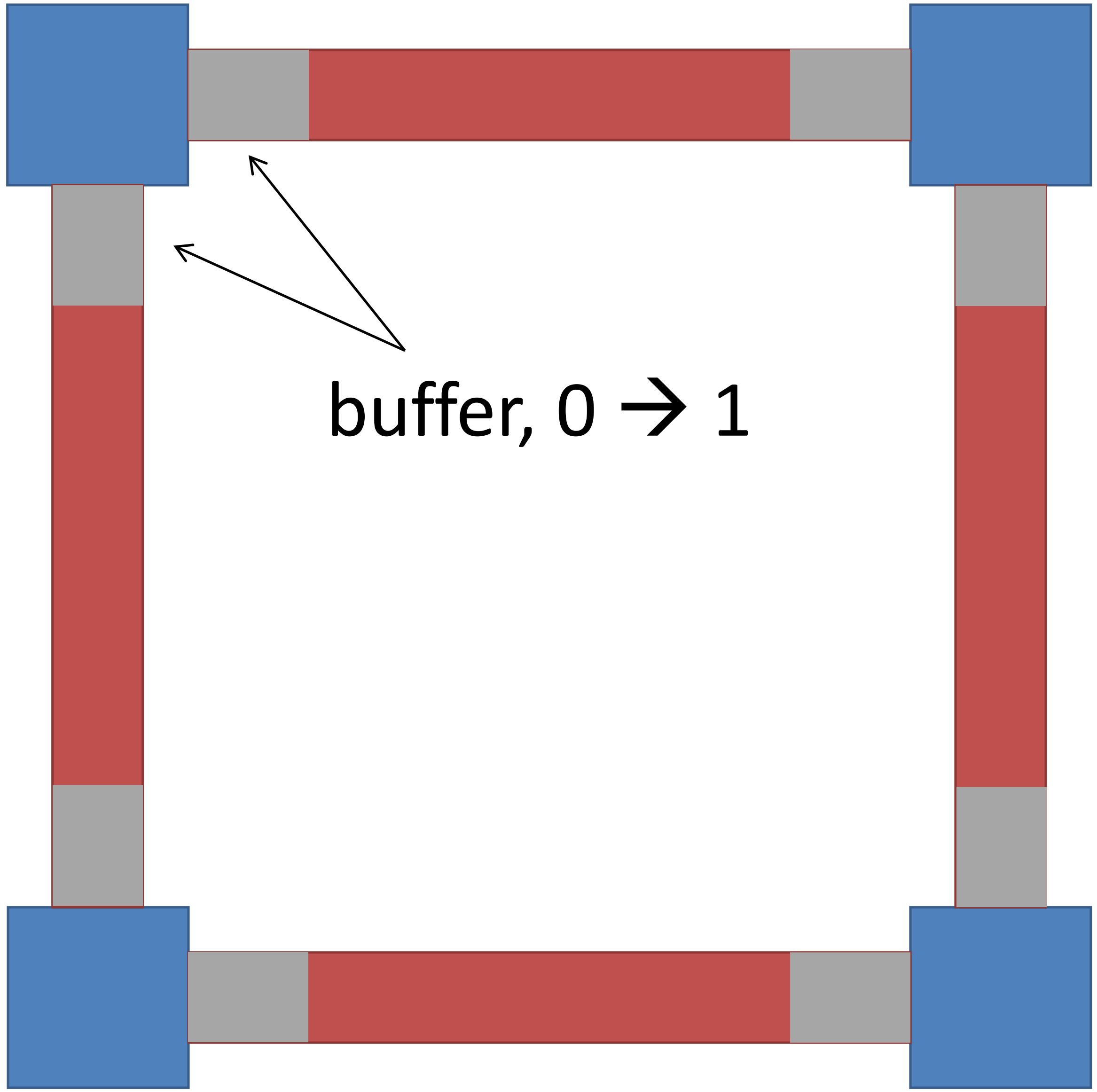
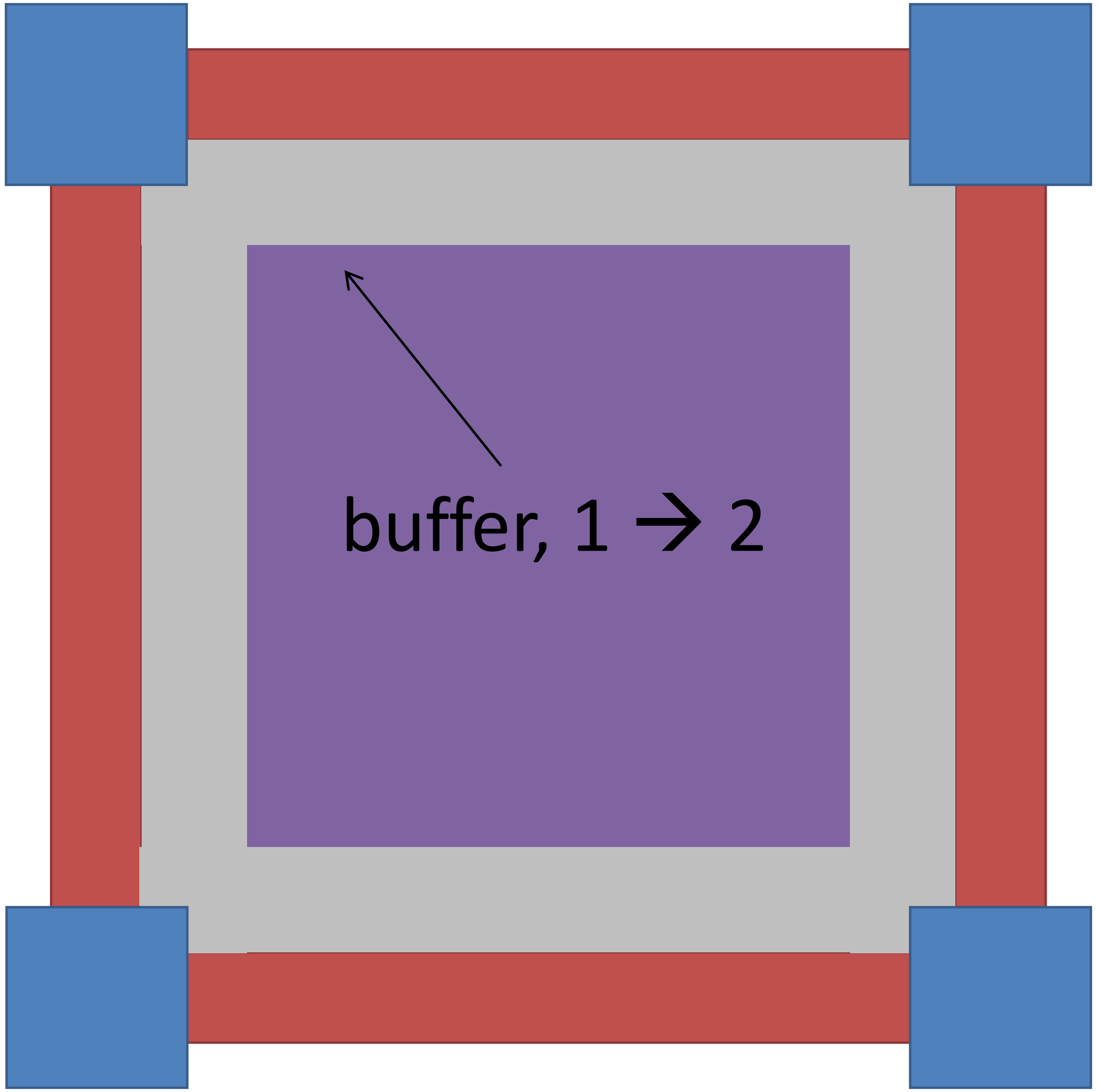
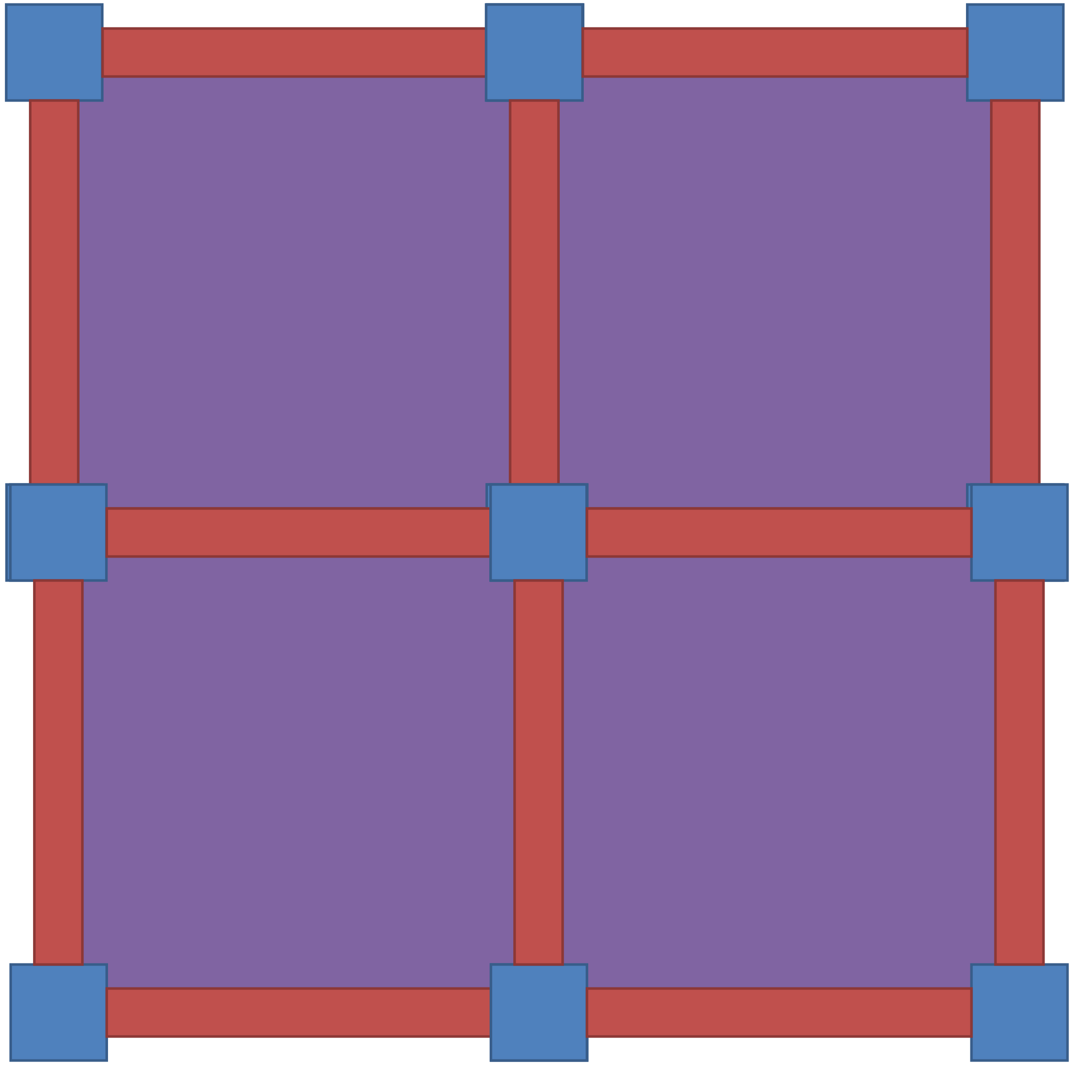
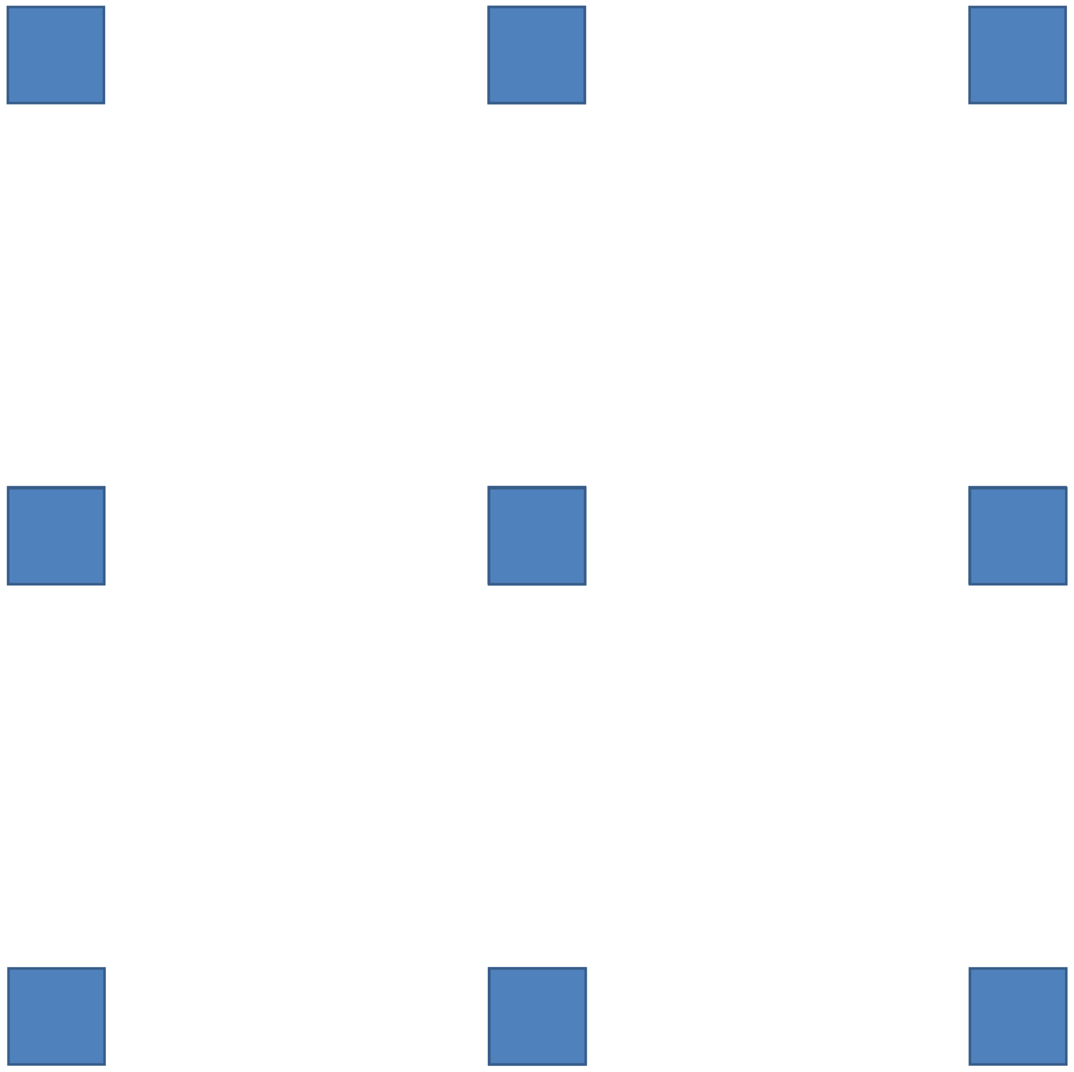
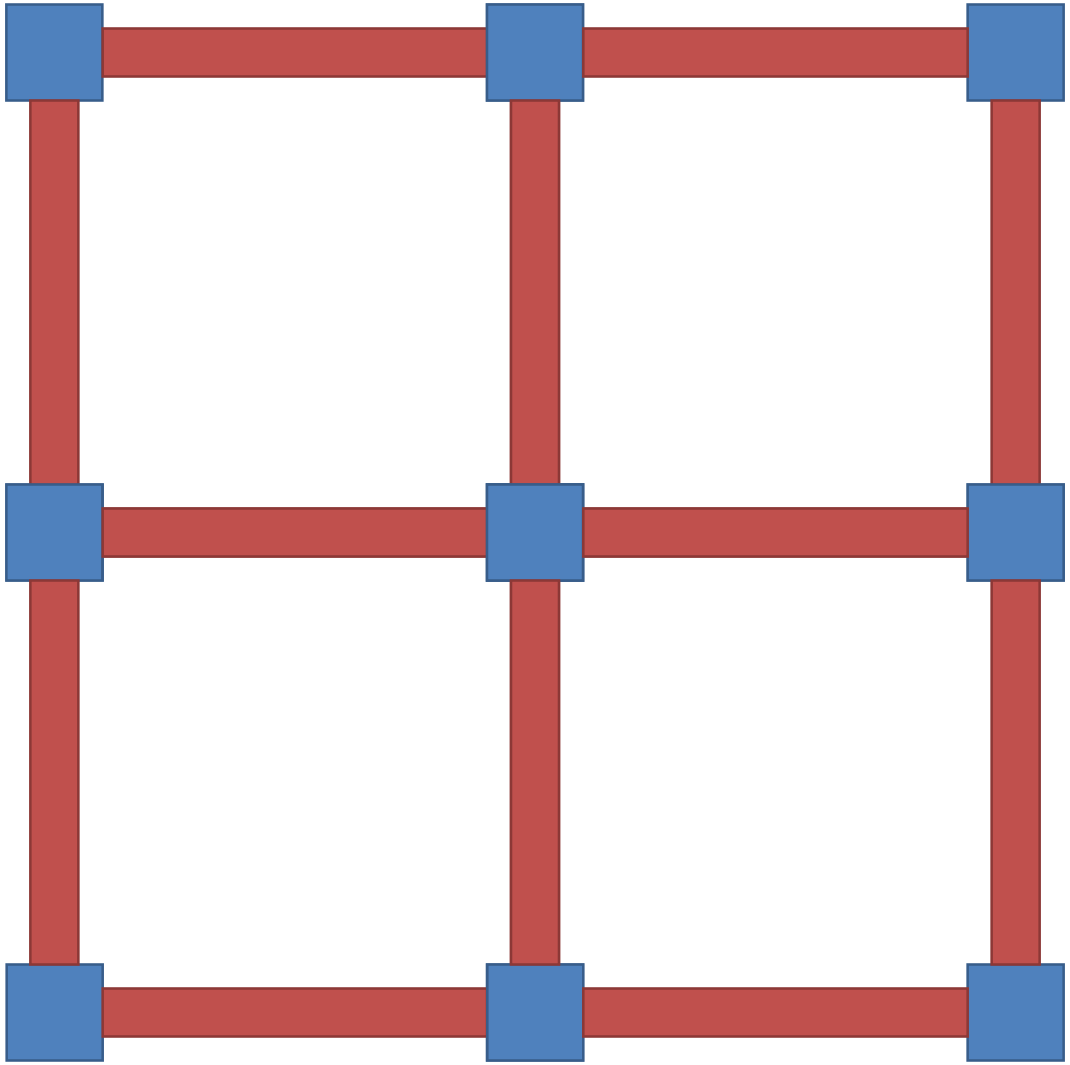

buffer, 0 → 1

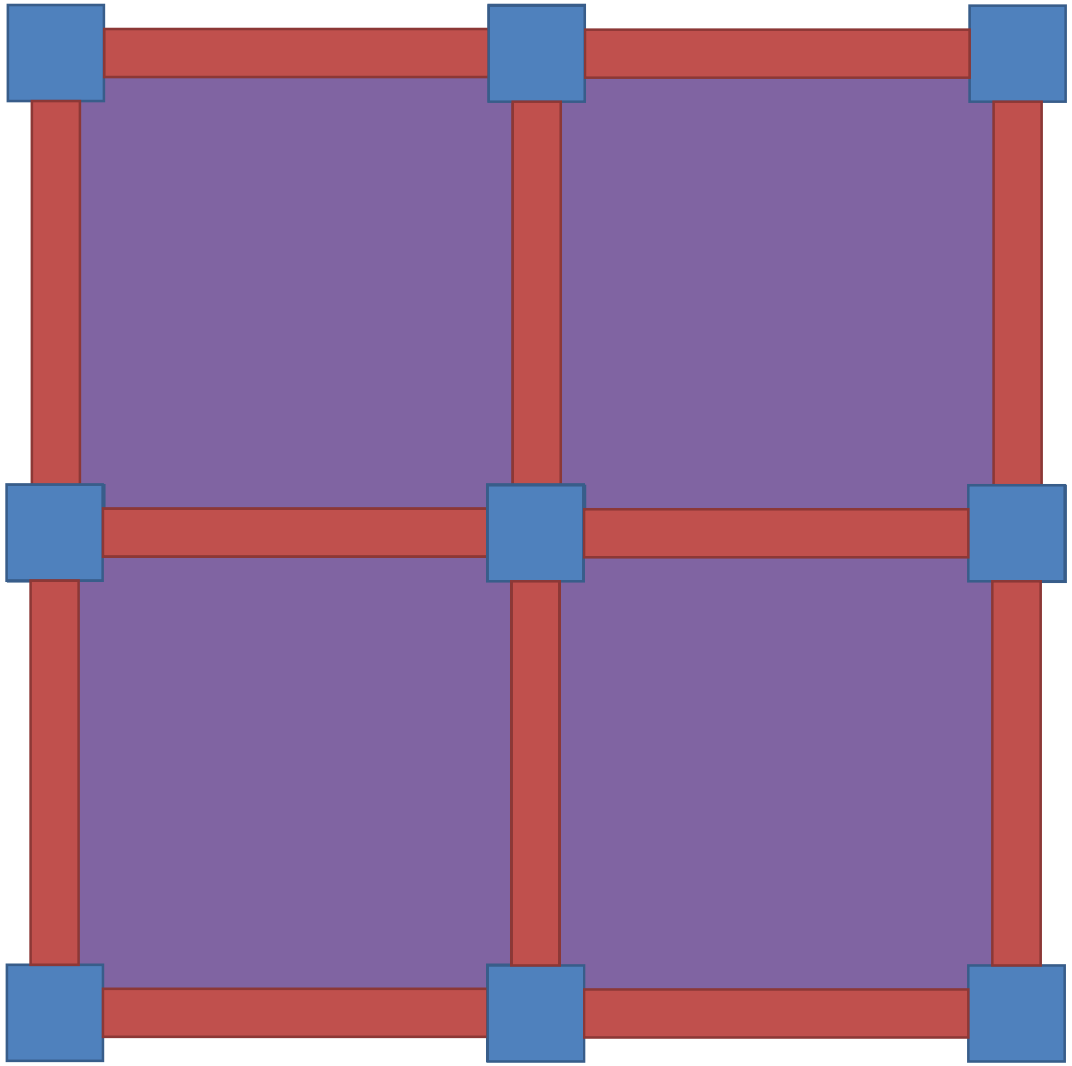

buffer, 1 → 2

# Tile the plane with these unit cells:









# Bubble of nothing array

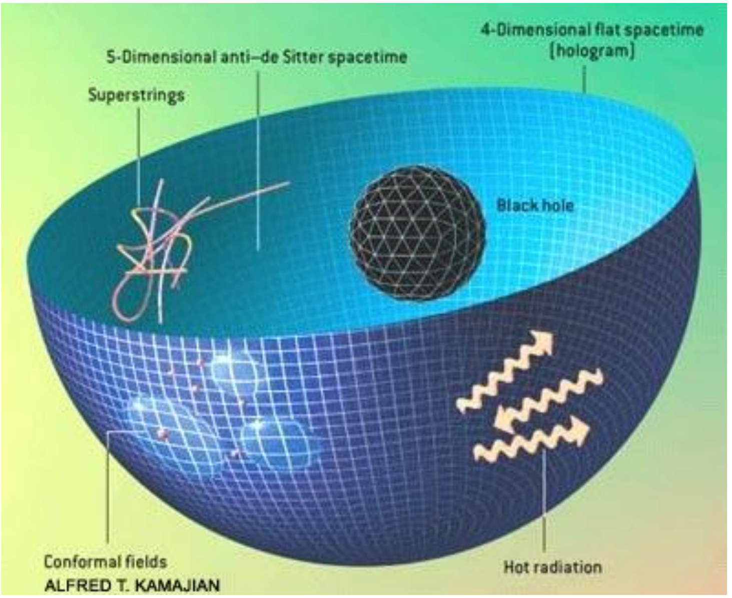
# Main result

- Given: a state of n qubits with vanishing CMI for appropriate regions
- Result: there exists a pure state on a doubled system which is (approximately) the unique gapped ground state of a local Hamiltonian and which is adiabatically connected to a product state

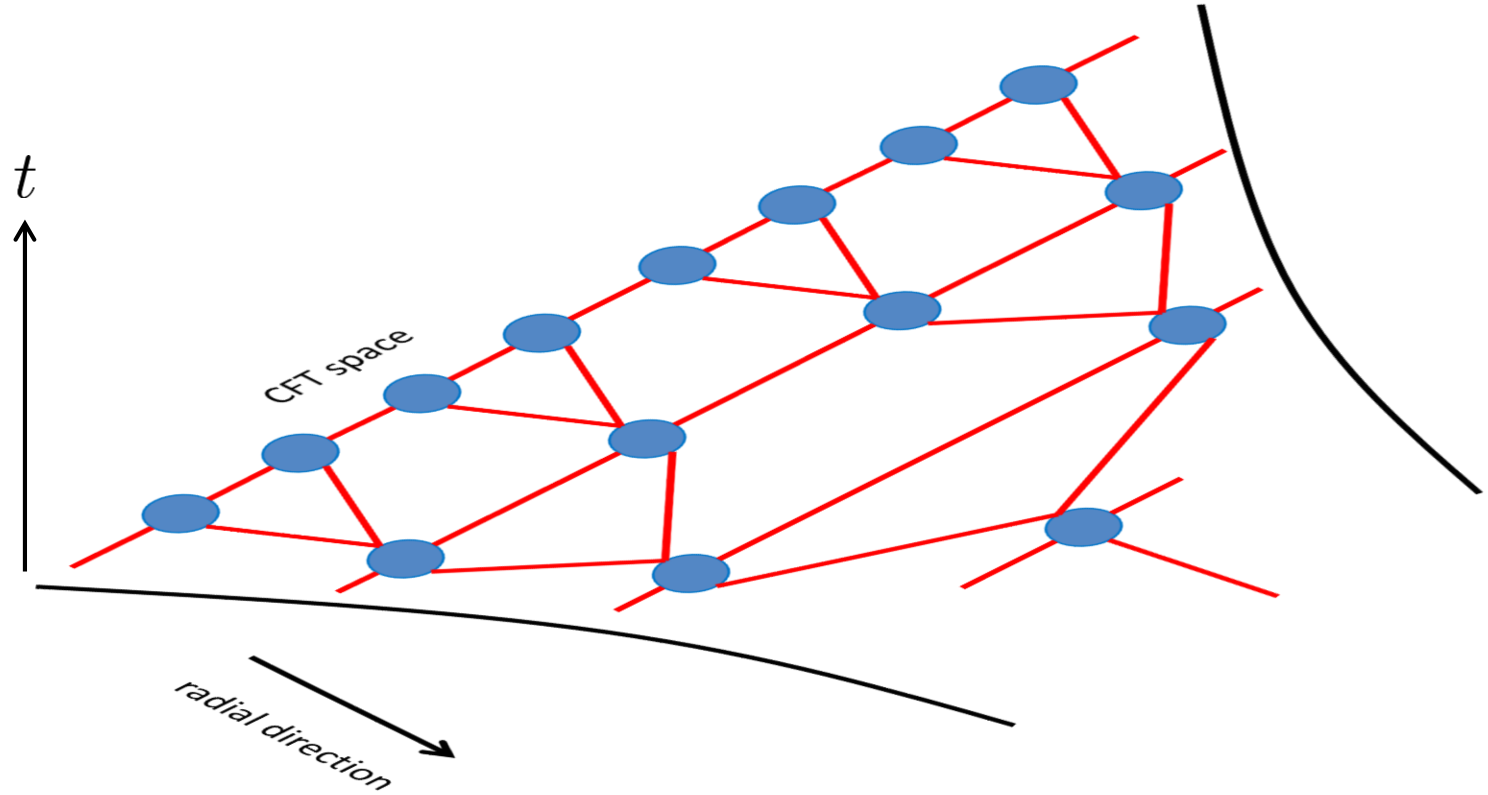
[BGS-McGreevy 1607.05753]

# Connections to geometry and gravity

- AdS/CFT or holographic duality asserts: certain quantum field theories *without gravity* are exactly equivalent to certain quantum gravities in a higher dimension [Maldacena]
- Emergent direction is associated with energy scale in the field theory, an "RG direction"

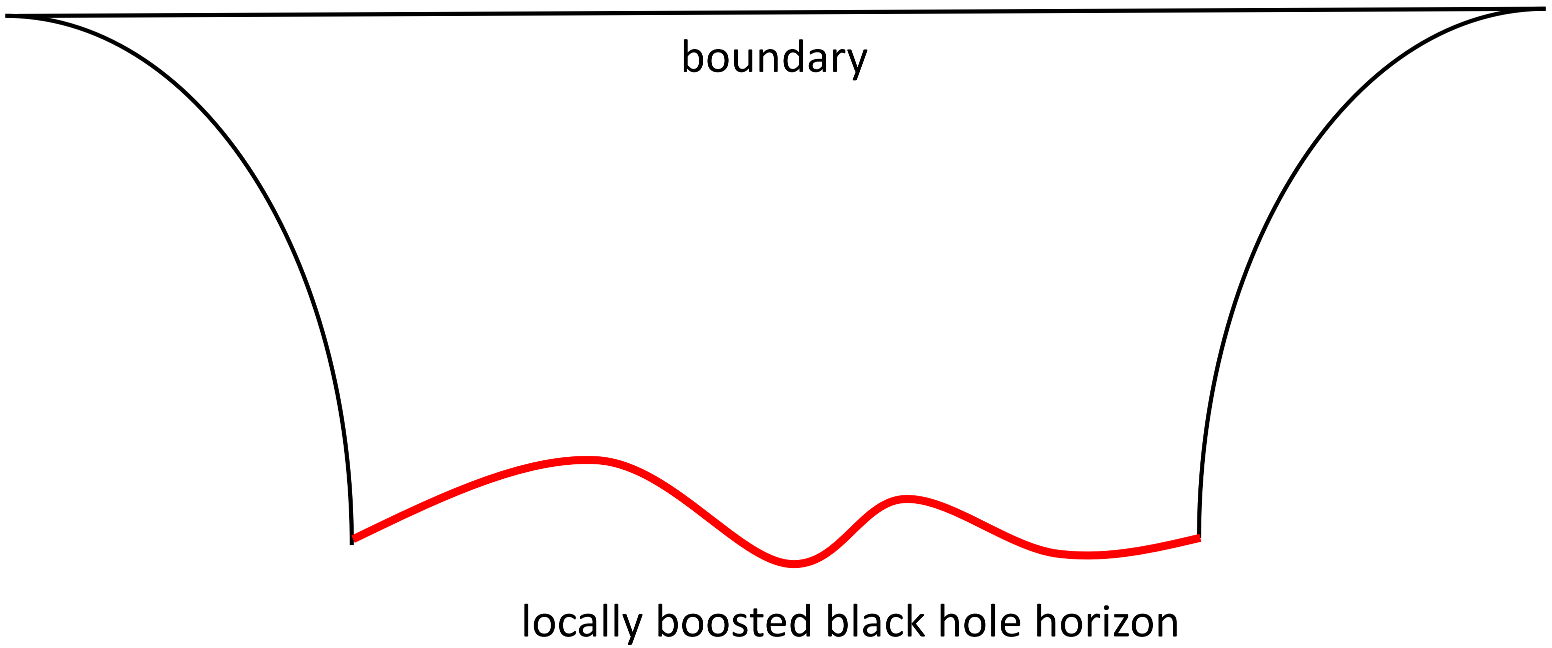



# Spacetime as a tensor network [BGS]




Scale invariant CFT ground state → $ds^2_{\mathrm{AdS}} = \frac{\ell^2}{r^2}(dr^2 - dt^2 + d\vec{x}^2)$

# Hydrodynamic states in AdS/CFT




[Bhattacharyya-Hubeny-Minwalla-Rangamani]

# Matching the tensor network and black hole descriptions

- AdS/CFT: thermal states of CFTs with holographic duals have approximately vanishing CMI  [BGS-McGreevy 1607.05753]
- Remains true for hydrodynamic CFT states dual to hydrodynamic black holes  [BGS-Hubeny coming soon]
- In both cases, we have a patch description of the state
  - Thermal scale chunks of the density matrix – local currents
  - Thermal scale patches of the black hole horizon – local boosts

# Summary and work-in-progress

- Main result: efficient quantum information based representation of thermal states for a wide variety of quantum phases of matter

- Does the representation have an RG structure on scales smaller than the thermal scale?
- Can we derive hydrodynamic-like equations for the representation?
- Can we use the representation to make new calculations in strongly interacting systems?

THANK YOU